LTCC Exam – Mathematical Biology

Open the gene-regulatory network code that we studied in class by going to:

https://github.com/philip-pearce/GeneRegulatory

1. Sketch the repressilator gene-regulatory network.
2. Let $P_i = m_i$. You should now have 3 equations for 3 variables. Write down the Jacobian matrix for the repressilator system of equations, in terms of the steady state values of the variables (you don't need to calculate these yet). It will be a 3x3 matrix.
3. Let $\alpha_0 = 0$. Write a function that takes in the parameters α and n and calculates the eigenvalues of the Jacobian matrix using Python (an example of how to calculate eigenvalues is given at the bottom of the online code). You can use the fact that at steady state we have $m_1 = m_2 = m_3 = m_0$,

   where

   $m_0 = \frac{\alpha}{1 + m_0^n}$.

   To solve for m_0 in this equation will require using a root-finding code, e.g. scipy.optimize.

   root (or use something else such as Mathematica). Always choose the root for which m_0 is real and positive.

   Now, using your code, scan through the parameter space for n = 2, n = 3, n = 4 and n = 5 and for α = 1, α = 10 and α = 100, and denote in a figure where you predict the steady solution will be linearly stable, and where the steady solution will be linearly unstable.
4. Perform a time-dependent simulation using the online simulation code for one value of the parameters in the "stable" region and one value of the parameters in the "unstable" region; the initial condition shouldn't matter too much, but if you like you can use your steady state with a small perturbation added (i.e. add a small number to the value of each variable). You can set b=1000 so that the protein dynamics are fast (as assumed above). Output a plot that shows how the concentration of the gene-regulatory proteins/mRNA evolve in time. the results should be plotted for long enough to clearly see whether the steady state is stable or unstable and any obvious features of the solution.

Please send the answers in PDF format to office@ltcc.ac.uk by the end of the day on 11 April if you are taking 1 or 2 LTCC exams or by the end of the day on 15 April if you are sitting 3 or more exams.