LTCC
"MORSE THEORY, TOPOLOGY AND ROBOTICS"
EXAM 2023 - 2024

EXAMINER: PROFESSOR MICHAEL FARBER

1.

(a) Give the definition of a topological manifold.

(b) Give the definition of a smooth manifold.

(c) What is meant by a 1-parameter group of diffeomorphisms,

(d) State the theorem about 1-parameter group of diffeomorphisms generated by a vector field.

2.

(a) Give the definition of a critical point;

(b) When do we say that a critical point is Morse?

(c) State the Morse Lemma.

(d) Find the critical points of the function
$$f(x,y) = x^3 - 3xy^2$$
where $(x, y) \in \mathbb{R}^2$. Decide if some of these critical points are non-degenerate.

(e) State the Morse inequalities.

(e) What is the minimal number of critical points of a Morse function on a closed orientable surface of genus g?

3.
  (a) Give the definition of a cyclic linkage.

  (b) Describe the configuration space of a planar linkage.

  (d) When do we say that a length vector is generic?

  (e) Decide if the length vector $\ell = (1, 1, 1, 1, 1)$ is generic.

  (f) What is the dimension of the moduli space M_ℓ of planar linkage with length vector $\ell = (1, 1, 1, 1, 1)$?

  (g) Using the general theorem given in the course, find the first Betti number $b_1(M_\ell)$ where $\ell = (1, 1, 1, 1, 1)$.