LTCC: Time Series Analysis
Mock Exam

Part I: R

Dataset

Download the data set that contains average monthly house prices in the London Region from January 1995 to December 2022 from the Land Registry website `http://landregistry.data.gov.uk/app/ukhpi/explore/`.

To do this, visit the Land Registry website and performing the following sequence of operations: customise your search → select location → London → Select dates → From 01/1995 (when data recording starts for this index) to 12/2022, and download as CSV.

In R, read in the data using the command `read.csv` and store the series to `p`. Some data pre-processing might be required; you can do it either directly in Excel or in R; it is also a good idea to take a look at the data file and check its format, before you start answering the questions.

Questions

1. Explain why it often makes sense to consider time series of prices on a logarithmic scale.

2. Now use the command `logp <- log(p)` to obtain the logged series. Denoting the series contained in `p` by P_t and the series contained in `logp` by L_t, plot both P_t and L_t and comment on their most obvious visual features. Do these time series appear stationary to you?

3. L_t displays a strong upward trend. As explained during the lectures, one way of eliminating trend is by differencing. Produce the following time series in R:

   - The series of first differences of L_t:
   $$U_t = L_t - L_{t-1}$$
   - The series of second differences of L_t (= first differences of the first differences):
   $$V_t = U_t - U_{t-1}.$$

Plot the time series U_t and V_t and comment on their visual appearance.

4. Produce the plots of sample autocorrelation functions AND sample partial autocorrelation functions for both U_t and V_t.

5. Based on what we have learned about the shapes of the acf and pacf for the different types of time series models, what ARMA(p, q) models would you initially consider for U_t and V_t? How about making use of AIC and BIC? Specify your chosen values of p and q.

6. A customer wishes to know your prediction for P_t for the first three months in 2023. Based on the data and on the above model, what is your best guess as to the house price P_t? Would you also give their confidence intervals?

Part II: Theory

1. Consider the following process

$$X_t = 0.5X_{t-1} + \epsilon_t - 1.4\ \epsilon_{t-1} + 0.45\ \epsilon_{t-2},$$

where ϵ_t is *i.i.d.* white noise with variance σ^2.

   (a) Argue that this is an ARMA(p, q) process, and specify the values of p and q.

   (b) Is it causal and/or invertible?

   (c) What is the ACF of this process?

2. For a stationary (both weakly and strongly) ARCH(1) process $\{X_t\}$ with $X_t = \sigma_t\epsilon_t$, $\sigma_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2$ and $\epsilon_t \overset{i.i.d.}{\sim}$ Uniform$[-\sqrt{3}, \sqrt{3}]$. Here we assume that $\alpha_0 > 0$, $\alpha_1 \in (0, 1)$ and $EX_t^6 < \infty$.

   (a) Derive EX_t, EX_t^2, EX_t^3, EX_t^4.

   (b) Verify that $\{\sigma_t^2(\epsilon_t^2 - 1)\}$ is a series of white noise.

   (c) Find out the ACVF of $\{X_t\}$ and $\{X_t^2\}$.

   (d) Is $\{X_t\}$ white noise? How about $\{X_t^2\}$?